Positive solutions with finite energy for a certain class of elliptic systems

Aleksandra Orpel

* Faculty of Mathematics and Computer Science, University of Lodz, Poland
E-mail: aleksandra.orpel@wmii.uni.lodz.pl

We investigate the following class of elliptic problems

$$\begin{cases} \triangle u_1(x) + f_1(x, u_1(x), u_2(x)) + g_1(x)x \cdot \nabla u_1(x) = 0 \\ \triangle u_2(x) + f_2(x, u_1(x), u_2(x)) + g_2(x)x \cdot \nabla u_2(x) = 0 \end{cases}$$

for $x \in G_R$, where $G_R = \{x \in \mathbb{R}^n, ||x|| > R\}$, $n > 2$, $R > 2$, with conditions

$$\lim_{||x|| \to +\infty} u_1(x) = 0, \quad \lim_{||x|| \to +\infty} u_2(x) = 0.$$

We consider the case when f_i and g_i, $i = 1, 2$, are sufficiently smooth and nonradial functions. The main goal of the presentation is to formulate sufficient conditions guaranteeing the existence of positive solutions with finite energy in a neighborhood of infinity. Our approach is based on subsolutions and supersolutions methods. Applying the Kawano's ideas and the Sattinger's monotone iteration procedure we construct two solutions (a maximal solution and a minimal solution) to our problem. We also discuss their behavior at the infinity. We also establish necessary conditions for the existence of this type of positive solutions of our system with a certain class of nonlinearities.

References

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