Singular elliptic problems - existence of soltuions and their properties

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We present our results regarding the existence of positive solutions of the following problem

$$\begin{cases} \Delta u(x) + f(x, u(x)) - b(x)(u(x))^{-\alpha}||\nabla u(x)||^{\beta} + g(x)x \cdot \nabla u(x) = 0, \\ \lim\limits_{||x||\to\infty} u(x) = 0, \end{cases} \tag{0.1}$$

where $n > 2$, $R > 1$, $||x|| := \sqrt{\sum\limits_{i=1}^{n} x_i^2}$, $\Omega_R = \{x \in \mathbb{R}^n, ||x|| > R\}, 0 < \alpha \leq \beta - \alpha$, which originates from the problem formulated in works of A. Constantin ([1]) without the singular part.

In the first part of the presentation, we consider a problem without a singular part and show the multiplicity of solutions. As our main tool we use the Noussair-Swanson theorem concerning the sub-supersolution ([3]).

In the second part, we show the existence of a classical solution to (0.1) using both sub-supersolution method ([2]) as well as the unbounded domain approximation method, described, among others in [3] and [4]. Here we use solutions from the first part as supersolutions of (0.1).

References

[1] A. Constantin, *Existence of positive solutions of quasilinear elliptic equations*, Bull. Austral. Math. Soc. **54** (1996) 147–154.

[2] Cui, S., *Existence and nonexitence of positive solutions of singular semilinear elliptic boundary value problems*, Nonlinear Analysis, 41, (2000), 149-176.

[3] E.S. Noussair, C.A. Swanson, *Positive solutions of quasilinear elliptic equations in exterior domains*, J. Math. Anal. Appl. **75** (1980) 121–133.

[4] A. Orpel, *Multiplicity of positive solutions for singular elliptic problems*, Mathematische Nachrichten, (2021) Vol. 294 no. 12