Recent Studies on Sobolev mappings

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A homeomorphism U of the unit disk $\mathbb{D} \subset \mathbb{R}^2$, $U = (u_1, u_2) : \mathbb{D} \to \mathbb{D}$ is a quasiharmonic map if $u_i \in W^{1,1}_{loc}$, $i = 1, 2$ are finite energy solutions to the system

$$\begin{cases} div(B(y)\nabla u_1) = 0 & a.e.\, in\, \mathbb{D} \\ div(B(y)\nabla u_2) = 0 & a.e.\, in\, \mathbb{D} \end{cases}$$

for a symmetric degenerate elliptic conductivity $B = B(y)$, i.e.

$$\frac{|\xi|^2}{H(y)} \leq \langle B(y)\xi, \xi\rangle H(y)|\xi|^2 \, a.e.\, in\, y \in \mathbb{D}\, \forall \xi \in \mathbb{R}^2$$

where $H : \mathbb{D} \to [1, \infty[$ is measurable. A sufficient condition that the Sobolev homeomorphism $U \in W^{1,1}_{loc}$ is a quasiharmonic map is that $U^{-1} \in W^{1,1}_{loc}$. This condition is not necessary because we construct quasiharmonic map such that $U^{-1} \in BV \setminus W^{1,1}_{loc}$