Inner variations and contours of images in variational denoising

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We consider minimization problem for a class of functionals of form

$$\mathcal{E}(w) = \int_\Omega \varrho(Dw) + \int_\Omega \psi(w - f),$$

where ϱ, ψ are convex and f is a given *noisy image*. In the case $\varrho = |\cdot|$, $\psi = |\cdot|^2/2$, this is known as the Rudin–Osher–Fatemi denoising model. The function ϱ is chosen to be of linear growth, so that minimizers do not have too high (e.g. Sobolev) regularity. Instead, they are BV functions, in particular they can have jump discontinuities which correspond to sharp contours of images.

Under mild assumptions on ϱ and ψ, we show that the jumps are contained in the jumps of f, provided that the latter is a bounded BV function. Moreover, we give estimates on the magnitude of the jumps. Our technique is based on taking suitable inner variations supported near a jump point. This significantly simplifies and generalizes previous results on the subject [1, 3]. In particular, our method works in the vector-valued setting (corresponding to color images).

We also consider the case where f is a generic L^∞ function. In this case we define weak notions of jump set and jump magnitude, for which an analogous result holds.

References

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