

## Normed Orlicz function spaces which can be quasi-renormed with easily calculable quasi-norms

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We will be interested in a widest possible class of Orlicz functions  $\Phi$  such that the easily calculable quasi-norm  $[f]_{\Phi,p} := \|f\|_E \left\{ I_{\Phi} \left( \frac{f}{\|f\|_E} \right) \right\}^{1/p}$  if  $f \neq 0$  and  $[f]_{\Phi,p} = 0$  if  $f = 0$ , on the Orlicz space  $L^{\Phi}(\Omega, \Sigma, \mu)$  generated by  $\Phi$ , is equivalent to the Luxemburg norm  $\|\cdot\|_{\Phi}$ . To do this, we will use a suitable  $\Delta_2$ -condition, lower and upper Simonenko indices  $p_S^{\alpha}(\Phi)$  and  $q_S^{\alpha}(\Phi)$  for the generating function  $\Phi$ , numbers  $p \in [1, p_S^{\alpha}(\Phi)]$  satisfying  $q_S^{\alpha}(\Phi) - p \leq 1$ , and an embedding of  $L^{\Phi}(\Omega, \Sigma, \mu)$  into a suitable Köthe function space  $E = E(\Omega, \Sigma, \mu)$ . We will take as  $E$  the Lebesgue spaces  $L^r(\Omega, \Sigma, \mu)$  with  $r \in [1, p_S^1(\Phi)]$ , when the measure  $\mu$  is non-atomic and finite, and the weighted Lebesgue spaces  $L_{\omega}^r(\Omega, \Sigma, \mu)$ , with  $r \in [1, p_S^{\alpha}(\Phi)]$  and a suitable weight function  $\omega$ , when the measure  $\mu$  is non-atomic infinite but  $\sigma$ -finite. We will also use condition  $\nabla_3$  if  $p_S^{\alpha}(\Phi) = 1$  and condition  $\nabla^2$  if  $p_S^{\alpha}(\Phi) > 1$ , proving their necessity in most of the considered cases.

We are inspired by two examples of easily calculable quasi-norms in Orlicz spaces equivalent to the Luxemburg norm, presented by Iwaniec and Verde in 1999, and by Krbeč and Schmeisser in 2012.

Our results seem important for applications of the Orlicz function spaces.

Based on:

P. Foralewski, [H. Hudzik](#), R. Kaczmarek and [M. Krbeč](#), "Normed Orlicz function spaces which can be quasi-renormed with easily calculable quasi-norms", Banach J. Math. Anal. 11 (2017), No. 3, 636–660.