Existence of positive solutions of a certain class of semipositone problems in exterior domain

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The aim of the work is to find sufficient conditions to prove that the equation

$$\begin{cases} \triangle u(x) + f(x, u(x)) + g(x, u) \cdot x \cdot \bigtriangledown u(x) = 0 \quad \text{for} \quad x \in \Omega_R \\ \lim\limits_{\|x\| \to +\infty} u(x) = 0 \end{cases}$$

has a classical positive solution in exterior domain $\Omega_R = \{x \in \mathbb{R}^n, \|x\| > R\}$ for some real number $R > 1$ and integer $n > 2$. We apply the subsolution-supersolution method based on Noussair-Swanson theorem as a main method to show the existence of a solution of our problem. We will construct the supersolution as a radial solution of some standard elliptic auxiliary problem. In this case we need to use the fixed point theorems. We also want to describe how the solution behaves at infinity. We divide our considerations in two parts - with f positive (*positone problem* - part I) and f negative (*semipositone problem* - part II) at the origin. In the second part we consider the case when function $g(x, u) = g(x) \cdot u^{q_2}$, where $q_2 > 1$ is some given number.

References

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